Instability of Centralized Markets[*]

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Abstract

Centralized markets reduce search for buyers and sellers. Their 'thickness' increases the chance of order execution at nearly competitive prices. In spite of the incentives to consolidate, some markets, securities markets and on-line advertising being the most notable, are fragmented into multiple trading venues. We argue that fragmentation is an inevitable feature of any centralized market except in special circumstances.[1]

1   Introduction

A centralized market reduces the costs of clearing, settlement and search compared to one consisting of multiple trading venues. Were these costs to decline because of technological innovation, a centralized market should still dominate a fragmented market because traders would prefer the venue that offers the highest probability of order execution and the most competitive prices. Each additional trader on an exchange reduces the execution risk for

other potential traders, attracting more traders. This positive feedback should encourage trade to be concentrated in a single exchange.

In spite of the incentives to consolidate, many markets have fragmented into multiple trading venues. Securities markets are the most well known example. They face a host of competitors such as ECNSs (electronic communication networks), ATSs (alternative trading systems) and the trading desks of broker-dealer firms. These alternative venues are not restricted to non-standardized assets and have large trading volumes. Some venues, like OTCs (over the counter) specialize in bilateral contracts. With corporate bonds, for example, traders approach counterparties sequentially. Offers are only good 'as long as the breath is warm,' which prevents obtaining multiple quotes before committing to trade (see Bessembinder and Maxwell (2008)). Other venues operate exchanges in which trade occurs at different prices on both the buy side and the sell side of the market. Securities markets are not unique in this respect. In on-line advertising there are 5 major exchanges that are 'open' and numerous others that are 'private'.[2] Madhavan (2000) calls this the *network externality puzzle* and writes: "Despite strong arguments for consolidation, many markets are fragmented and remain so for long periods of time."

A variety of reasons (not entirely mutually exclusive), summarized below, have been offered for why centralized markets fragment.

1. Competition: Fragmentation enhances efficiency because competition between exchanges forces them to narrow their bid-ask spreads (e.g., Pagano (1989); Biais, Martimort, and Rochet (2000)). Fragmentation in securities markets has been associated with regulation designed to limit the abuse of market power by operators of centralized exchanges.[3] Fragmentation can also enhance efficiency (total welfare) by limiting the market power of participants (Malamud and Rostek (2017)).[4]

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[2]For fragmentation in labor markets see Roth and Xing (1994).

[3]Regulation National Market System in the US and the Market in Financial Instruments Directive (MiFID) in Europe. `http://www.sechistorical.org/museum/galleries/msr/msr04b_increments_ecns.php`

[4]Malamud and Rostek (2017) provide examples where a market fracture can increases total welfare of

2. Heterogenous Preferences: Alternative trading venues arise to cater to traders who differ in their preferences for order size, anonymity and likelihood of execution (Harris (1993), Ambrus and Argenziano (2009) and Petrella (2009)).

3. Informational: Traders seek out alternative venues so as to conceal private information (see Madhavan (1995)), other venues spring up to attract uninformed traders from the incumbent exchange (Easley, Kiefer, and O'Hara (1996)) or competing venues affect the incentives to acquire information (Glode and Opp (2016)).

4. Congestion: As a market becomes thicker, the time to evaluate and select offers lengthens, during which time prices may change. This encourages participants to transact earlier, fragmenting the market in time (see Roth and Xing (1994)).[5]

Congestion is unavoidable as it takes time to communicate and aggregate preferences. Even if one were to reduce congestion, the first three reasons remain. However, even these could be obviated, in principle, through the use of a suitable (possibly impractical) mechanism. In the first case, the intermediary could be mandated to implement the constrained efficient mechanism. In the second and third case, allowing agents to use a mechanism that employs a richer message space to communicate preferences and private information could be employed.

In this paper we argue that the instability of multilateral trading mechanisms is another force towards fragmentation. Within the model in which we make this point, the reasons for fragmentation just enumerated don't apply nor does it rely on the institutional details of either securities or advertising. It is the model of trade (Myerson and Satterthwaite (1983)) where each seller has one unit of a homogeneous good and each buyer is interested in purchasing at most one unit of the same good. The private type of each buyer is their marginal value for the good and the private type of each seller is the opportunity cost of their endowment. Thus, agents are all interested in the same order size. Holding prices equal,

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market participants, however, the payoff of some agents may be lower post fracture. We focus on the incentives for a group of agents to break off from the centralized market.

[5]Congestion can cause fragmented markets to persist as agents tradeoff thickness in one venue for less competition in another (Ellison and Fudenberg (2003), Ellison, Fudenberg and Mobius (2004)).

they are indifferent about who they trade with. There is no common-values component in the private information of agents, making them equally informed (or uninformed). Trade takes place in a single time period, so the timing of trades is irrelevant.

What makes a market centralized? If there are numerous distinct trading venues but an agent can simultaneously participate in all of them, we view this as a centralized market. Now suppose an agent may participate in at most one of the venues. If the law of one price prevails, we view this as a centralized market as well. Hence, we model a centralized market as a trading mechanism offered by an intermediary with no private information (called the incumbent mechanism) that is open to any agent. Trade within the incumbent centralized market is modeled as being conducted via an individually rational, weakly budget balanced and incentive compatible mechanism. The first condition is true of almost all observed trading mechanisms. The second prevents the operator of the incumbent exchange from subsidizing trades. The third recognizes that trading agents will act strategically, i.e., violate the price-taking assumption. Even if the mechanism in the incumbent exchange were not incentive compatible, by the revelation principle there would be a corresponding incentive compatible direct mechanism that replicates the outcome of the mechanism in the incumbent exchange.

A centralized market is stable if no subset of agents has an incentive to deviate and trade among themselves using a different mechanism, called the blocking mechanism. The blocking mechanism differs from the incumbent mechanism in that it is permitted to restrict participation. Our message is that budget balance and incentive compatibility conspire to make stability difficult if not impossible to achieve. Given that individual rationality, incentive compatibility, budget balance and efficiency are incompatible, we interpret this to mean that it is the violation of the price taking assumption that makes a centralized market vulnerable to fragmentation. This may appear obvious. Lack of price-taking should result in inefficiency, leaving gains from trade on the table. These gains could be secured by a group of agents leaving the centralized market to trade among themselves. This begs the question

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of why the departing agents can overcome the incentive problems associated with a lack of price-taking but the centralized market can't.

Formalizing the idea of blocking presents two conceptual difficulties. First, the decision to participate in the blocking mechanism conveys information about one's type which alter the beliefs of potential counterparties. Second, payoffs in the incumbent mechanism depend on the equilibrium played in that mechanism, which can be affected by the presence of a blocking mechanism. The notion of blocking we employ, due to Peivandi (2013), assumes that agents in the incumbent mechanism play a dominant strategy equilibrium. To emphasize the connection to dominant strategy equilibrium we call it D-blocking. D-blocking differs from prior notions of blocking used in the theory of cooperative games by allowing agents to condition their beliefs about counterparties based on which mechanism they are participating in. Roughly, an incumbent mechanism is *blocked* by a coalition of agents and a blocking mechanism if the blocking mechanism gives to each member of the blocking coalition, for a critical subset of their types, at least as much surplus as they would obtain if they remained in the incumbent mechanism. Furthermore, no agent with a type outside of their critical subset of types will participate in the blocking mechanism. We argue that the conditions under which a mechanism is immune to blocking are restrictive. From this, we conclude that centralized markets are unstable.

We restrict attention to deterministic mechanisms that are ex-post individually rational, ex-post (weakly) budget balanced (EBB) and dominant strategy incentive compatible (DSIC). The DSIC property is usually justified on two grounds. First, it appears to capture the spirit of 'safety' advanced by Roth (2007) as an important desidaratum of a centralized market. Second, it ensures that the mechanism is robust to the beliefs of agents (see Hagerty and Rogerson (1987)). DSIC does not exclude the possibility that the mechanism can depend on the designer's beliefs. For example, the designer could select a single price at which all trade must take place *a priori,* which depends on the designer's beliefs about the distribution of types of the agents. We show that almost any ex-post individually rational,

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EBB, and DSIC mechanism can be "D-blocked" by another ex-post individually rational, EBB, and DSIC mechanism.

D-blocking is accomplished by a particularly simple mechanism called a non-negative spread posted price mechanism. In this mechanism, two prices $p_1 \leq p_2$ are posted. If buyer and seller agree to trade, the seller is paid p_1 and the buyer pays p_2. The spread of a non-negative spread posted price mechanism is $p_2 - p_1$, and this is what the designer pockets. Thus, every ex-post individually rational, EBB, and DSIC mechanism can be D-blocked by a mechanism that gives the operator of the blocking mechanism non-negative expected profit. In fact, the blocking mechanism can be implemented with one of the agents making a take it or leave it offer to a subset of agents. It succeeds by trading the probability of trade against margin. Relatively low type sellers will accept a lower margin for a higher probability of trade. Relatively high type buyers will accept a lower surplus for a higher probability of trade.

Our first result (Theorem 2) identifies a joint condition on the primitives and *bilateral* trading mechanisms that characterizes immunity to to D-blocking. Immunity requires the incumbent mechanism to depend on the type distribution and that the type distributions have the right 'curvature'. However, when trade is multilateral, Theorem 4 shows that for all positive density type distributions any mechanism whose ex-post payoffs are continuous in type is D-blocked. Taken together, these results imply that immunity to D-blocking requires that the incumbent mechanism be very sensitive to the details of the environment which is rarely true of observed trading mechanisms.

The next section introduces notation and a precise definition of D-blocking. The subsequent section states and proves the main results concerning D-blocking. We contrast D-blocking with prior notions of the core of games with incomplete information in section 4. Section 5 concludes with a discussion of our results and directions for future research.

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2 D-blocking

Let N_b and N_s be the set of buyers and sellers respectively and $N = N_b \cup N_s$. Each seller is endowed with one unit of the good. The opportunity cost of seller i for a unit of the good is $c_i \in C_i$ where $C_i \subset \mathbb{R}^+$ is bounded. The value of buyer i for a unit of the good is $v_i \in V_i$ where $V_i \subset \mathbb{R}^+$ is bounded. Valuations and opportunity costs are private information and independently distributed. Preferences are quasilinear; that is, buyer (seller) i's payoff from receiving (giving up) a quantity q of the good (interpret as probability) for a monetary payment (compensation) of t is $qv_i - t$ $(t - qv_i)$.

A direct mechanism is defined by an allocation rule and a payment rule. The allocation rule maps profiles of reports of agents' private information to an allocation. If Q is the allocation rule, denote the component of Q that corresponds to buyer i's (or seller i's) allocation by q_i^b(or q_i^s). Thus, $q_i^k : \prod_{i \in N_b} V_i \times \prod_{j \in N_s} C_j \to \mathbb{R}$ for $k \in \{b, s\}$. We require that an allocation rule be feasible in the sense that for all buyers $i \in N_b$ and all sellers $j \in N_s$ and all profiles $(v, c) \in \prod_{i \in N} V_i \times \prod_{j \in N_s} C_j$; $1 \geq q_i^b, q_i^s \geq 0$ and $\sum_{i \in N_b} q_i^b(v, c) = \sum_{i \in N_s} q_i^s(v, c)$.

The payment rule maps each profile $(v, c) \in \prod_{i \in N} V_i \times \prod_{j \in N_s} C_j$ to a *per-unit* price each agent must pay. If P is the payment rule, the component of P that corresponds to buyer i's (seller i's) per-unit payment is denoted p_i^b (or p_i^s). Thus, $p_i^k : \prod_{i \in N} V_i \times \prod_{j \in N_s} C_j \to \mathbb{R}^+$ for $k \in \{b, s\}$.

We now define dominant strategy incentive compatibility (DSIC). Let $v = (v_i, v_{-i})$, $c = (c_j, c_{-j})$, $\hat{v} = (\hat{v}_i, v_{-i})$ and $\hat{c} = (\hat{c}_j, c_{-j})$ be profiles of valuations and opportunity costs. The profile $\hat{v}$ differs from v in that only buyer i changes the report of his marginal value. The profile $\hat{c}$ differs from c in that only seller j changes the report of his opportunity cost. The mechanism (Q, P) is DSIC if for all v, $\hat{v}$, c and $\hat{c}$,

$$q_i^b(v, c)(v_i - p_i^b(v, c)) \geq q_i^b(\hat{v}, c)(v_i - p_i^b(\hat{v}, c))$$

and

$$q_j^s(v,c)(p_j^s(v,c) - c_j) \geq q_i(v,\hat{c})(p_i(v,\hat{c}) - \hat{c}_j).$$

Mechanism (Q, P) is ex-post individually rational (EIR) if for all buyer i and seller j, $q_i^b(v,c)(v_i - p_i^b(v,c)) \geq 0$ and $q_j^s(v,c)(p_j^s(v,c) - c_j) \geq 0$, for all profiles $(v,c) \in \prod_{i \in N_b} V_i \times \prod_{j \in N_s} C_j$.

Mechanism (Q, P) is ex-post budget balanced (EBB) if for all $(v,c) \in \prod_{i \in N_b} V_i \times \prod_{j \in N_s} C_j$,

$$\sum_{i \in N_b} p_i^b(v,c) q_i^b(v,c) - \sum_{j \in N_s} p_j^s(v,c) q_j^s(v,c) \geq 0.$$

In mechanism (Q, P), the utility that buyer $i \in N_b$ and seller $j \in N_s$ under profile (v,c) enjoy are $u_i^b((v,c), Q, P) = q_i^b(v,c)(v_i - p_i^b(v,c))$ and $u_j^s((v,c), Q, P) = q_j^s(v,c)(p_j^b(v,c) - c_j)$, respectively. The expected utility that buyer $i \in N_b$ with valuation v_i and seller $j \in N_s$ with opportunity cost c_j enjoy are $E_{v_{-i},c}[u_i^b((v_i, v_{-i}, c), Q, P)]$ and $E_{v,c_{-j}}[u_j^s((v_i, v_{-i}, c), Q, P)]$, respectively.

Now suppose an alternative feasible, ex-post individually rational, DSIC, EIR mechanism $(\hat{Q}, \hat{P})$:

$$\hat{p}_i^k : \prod_{i \in N_b} V_i \times \prod_{j \in N_s} C_j \to \mathbb{R}^+, i \in N_k, \ \hat{q}_i^k : \prod_{i \in N_b} V_i \times \prod_{j \in N_s} C_j \to \mathbb{R}, \ k \in \{b, s\}, i \in N_k$$

We give a definition of what it means for $(\hat{Q}, \hat{P})$ to D-block the incumbent mechanism (Q, P) by subsets $A_b \subseteq N_b$ of buyers and $A_s \subseteq N_s$ of sellers. Imagine that before participating in the mechanism (Q, P), each agent in $A = A_b \cup A_s$ (and only A) is invited to participate in $(\hat{Q}, \hat{P})$. If at least one of the agents in A declines the invitation, all agents are required to participate in (Q, P); in this case we say the D-block fails. If every agent in A accepts the invitation, this becomes common knowledge among them, and they enjoy the outcome delivered by $(\hat{Q}, \hat{P})$. The agents now face a Bayesian game in which each must first decide which of the two mechanisms to participate in and subsequently what to report in their

chosen mechanism. As each mechanism is DSIC, we assume truthful reporting. We say (Q, P) is D-blocked by the set A if there is a Bayesian equilibrium of the game where, with positive probability, all agents in A choose $(\hat{Q}, \hat{P})$. Formally, we need for each $i \in A_b$ (and $j \in A_s$) a positive measure subset $V_i' \subseteq V_i$ (and $C_j' \subseteq C_j$) and an equilibrium where each $i \in A_b$ and $j \in A_s$ chooses $(\hat{Q}, \hat{P})$ if their type is in V_i' and C_j', respectively, and (Q, P) otherwise. Call V_i' and C_j' the **critical** set of types for buyer i and seller j. For each $i \in A_b$ (and $j \in A_s$) let T_i^b (and T_j^s) be the event that the type of each agent $k \in A \setminus \{i\}$ is in their critical set. (Q, P) is D-blocked by the set A with respect to $\Pi_{i \in A_b} V_i' \times \prod_{j \in A_s} C_j'$ if the five conditions listed below hold.

1. For all buyers $i \in A_b$, if $v_i \in V_i'$, then,

$$E_{-i}[u_i^b((v_i, v_{-i}, c), Q, P)|T_i^b] \leq E_{-i}[u_i^b((v_i, v_{A_b \setminus \{i\}}, c_{A_s}), \hat{Q}, \hat{P})|T_i^b], \qquad (1)$$

and for all sellers $j \in A_s$, if $c_j \in C_j'$, then,

$$E_{-j}[u_j^s((v, c_j, c_{-j}), Q, P)|T_j^s] \leq E_{-j}[u_i^s((v_{A_b}, c_j, c_{A_s \setminus \{j\}}\}, \hat{Q}, \hat{P})|T_j^s]. \qquad (2)$$

2. For all buyers $i \in A_b$, if $v_i \notin V_i'$, then,

$$E_{-i}[u_i^b((v_i, v_{-i}, c), Q, P)|T_i^b] \geq E_{-i}[u_i^b((v_i, v_{A_b \setminus \{i\}}, c_{A_s}), \hat{Q}, \hat{P})|T_i^b], \qquad (3)$$

and for all sellers $j \in A_s$, if $c_j \notin C_j'$, then,

$$E_{-j}[u_j^s((v, c_j, c_{-j}), Q, P)|T_j^s] \geq E_{-j}[u_i^s((v_{A_b}, c_j, c_{A_s \setminus \{j\}}\}, \hat{Q}, \hat{P})|T_j^s]. \qquad (4)$$

In equations (1) and (3) the expectation is with respect to the types of all buyers and sellers except for buyer i. In equations (2) and (4) the expectation is with respect to the types of all buyers and sellers except for seller j.

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3. For all $(v, c) \in \prod_{i \in A_b} V_i' \times \prod_{j \in A_s} C_j'$:

$$\sum_{i \in A_b} \hat{q}_i^b(v, c) = \sum_{j \in A_s} \hat{q}_j^s(v, c). \tag{5}$$

4.

$$\forall (v, c) \in \prod_{i \in A_b} V_i' \times \prod_{j \in A_s} C_j' \quad \sum_{i \in A_b} \hat{q}_i^b(v, c) \hat{p}_i^b(v, c) \geq \sum_{j \in A_s} \hat{q}_j^s(v, c) \hat{p}_j^s(v, c). \tag{6}$$

5.

$$E[\sum_{i \in A_b} \hat{q}_i^b(v, c) \hat{p}_i^b(v, c) - \sum_{j \in A_s} \hat{q}_j^s(v, c) \hat{p}_j^s(v, c) | (v, c) \in \prod_{i \in A_b} V_i' \times \prod_{j \in A_s} C_j'] > 0. \tag{7}$$

Condition (1) states that if each $i \in A$ has a type in their critical set, then every agent in A choosing to participate in $(\hat{Q}, \hat{P})$ is a best response to the other agents in A doing so. Condition (2) states that if $i \in A$ is the only member with a type not in his critical set, then choosing to participate in (Q, P) is a best response for agent i. Condition (3) ensures demand matches supply. Condition (4) states that the mechanism is weakly ex-post budget balanced. Condition (5) requires that, on some profile, the D-blocking mechanism generates a positive surplus. There is a technical and a substantive reason for this condition. The strict inequality in (7) means that there is a strict incentive for someone to offer the D-blocking mechanism. In other notions of blocking, the analogue of inequality (1) holds strictly for some agent i to prevent a mechanism from blocking itself. We eliminate the possibility of an exactly budget balanced mechanism being D-blocked by itself with condition (7).

We assume that if any agent in A declines the invitation, all agents must participate in the incumbent mechanism. This makes D-blocking harder. To see why, suppose one buyer and one seller only. If any agent who accepts the invitation must trade in the alternative mechanism, there would be two pure strategy equilibria: one where both agents always choose the incumbent mechanism and one where both always choose the alternative. We also assumed that once the agents choose the blocking mechanism, and this becomes common knowledge,

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the choice is irrevocable. This is not essential because the incumbent mechanism is DSIC. Allowing agents to return to the incumbent mechanism after observing the participants in the blocking mechanism does not alter subsequent results.

3 Vulnerability to D-blocking

Our analysis will divide into the case of bilateral and then multilateral trade.

3.1 Bilateral Trade

Consider the case of one buyer and one seller. The set A of agents that could possibly D-block an incumbent mechanism is the set of all agents. We characterize the incumbent mechanisms immune to D-blocking. We also give a sufficient condition on the distribution of types such that the only ex-post individually rational, ex-post budget balanced and DSIC mechanism immune to D-blocking is a posted price mechanism. D-blocking is accomplished using a non-negative spread mechanism.[6] Kuzmic and Steg (2016) give a characterization of such mechanisms.

We assume types of agents are distributed according to a continuous distribution in the $[0, 1]$ interval with positive density. When there are two agents, let $c \in [0, 1]$ be the opportunity cost of the seller and $v \in [0, 1]$ be the valuation of the buyer. Assume c and v are private information distributed independently with density functions $g(c)$ and $f(v)$, respectively. Denote the corresponding cumulative distribution functions by G and F.

A positive spread posted price mechanism can always be D-blocked by a positive spread posted price mechanism with a smaller spread. Thus, the only mechanism (within the class considered) that may be immune to D-blocking is a posted price. To illustrate how a posted price might be blocked consider Figure 1.

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[6]The price the buyer pays is no less than the price the seller receives.

Figure 1: Blocking a posted price mechanism (left) by a positive-spread posted price mechanism (right).
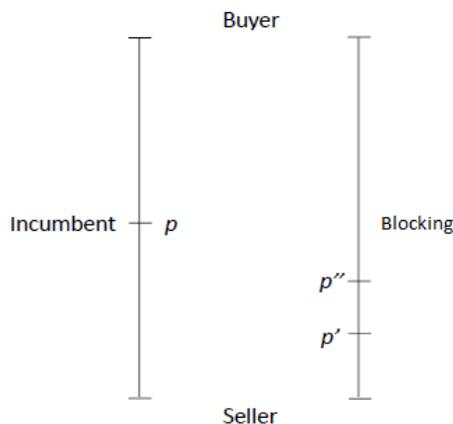
Buyer and seller types are located on a vertical line with the highest buyer and seller type at the top and the lowest buyer and seller type at the bottom. The vertical line on the left is associated with the incumbent mechanism employing a posted price of p. On the right is a vertical line associated with the putative blocking mechanism which employs a positive spread posted price with the pair (p', p''). The relative relationship of these prices are as displayed.

Any buyer with a type that exceeds p'' is, in expectation, strictly better off from choosing the blocking mechanism. Either they pay a price lower than p or they trade in the incumbent mechanism if the seller stays put. Why would a seller choose the blocking mechanism? If trade occurs in the blocking mechanism, they receive a lower price than in the incumbent mechanism. The blocking mechanism, however, offers a higher probability of trade. Buyers with types in $[p, p']$ who don't trade in the incumbent mechanism will participate in the blocking mechanism. Therefore, a seller earns a smaller margin in the blocking mechanism but a higher probability of trade. It is possible, then, for the expected profit for a seller whose type is small enough to enjoy a larger profit in the blocking mechanism. The next result shows that if we restrict the curvature of the distribution of types (i.e., elasticities of demand and supply) this cannot happen.

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Theorem 1 *If $x(1 - F(x))$ and $(1 - x)G(x)$ are single peaked[7] and $\arg\max_{x \in [0,1]} x(1 - F(x)) \geq \arg\max_{x \in [0,1]} (1 - x)G(x)$, then any posted price mechanism with a price*

$$p \in [\arg\max_{x \in [0,1]} (1 - x)G(x), \arg\max_{x \in [0,1]} x(1 - F(x))]$$

is immune to D-blocking.

The left-hand endpoint of this interval is the optimal monopsony price set by the highest type buyer. The right-hand endpoint is the optimal monopoly price set by the lowest type seller.

Proof. Let $p \in [\arg\max_{x \in [0,1]}(1 - x)G(x), \arg\max_{x \in [0,1]} x(1 - F(x))]$ be the posted price of the incumbent mechanism. Consider a positive spread posted price mechanism (p', p'') with $p' < p''$ as a possible D-blocking mechanism. As there are only two agents (one buyer and one seller), a D-blocking coalition will consist of just these two agents. It remains to identify a critical set of types. There are three cases:

1. **Case 1:** $p' < p'' \leq p$: A buyer with type $v \geq p''$ prefers the D-blocking mechanism conditioned on a seller being present. Thus, the critical set of types of the buyer will be $[p'', 1]$. Next, we find the critical set of types for the seller that would make them prefer the D-blocking mechanism. A seller with type $c < p'$ will join the D-blocking mechanism only if:

$$(p - c)Pr(v \geq p | v \geq p'') \leq (p' - c) \implies \frac{1 - F(p)}{1 - F(p'')} \leq \frac{p' - c}{p - c}.$$

   The right-hand side is maximized at $c = 0$; therefore, the posted price p *cannot* be D-blocked by the positive spread posted price mechanism (p', p'') if $\frac{1 - F(p)}{1 - F(p'')} > \frac{p'}{p}$. Therefore, if for all $p' < p$, $p(1 - F(p)) > p'(1 - F(p'))$, the posted price mechanism cannot be D-blocked with prices lower than p. This is clearly true given the choice of p and the

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[7]A function h is single peaked in the interval l $[a, b]$ if there exists $x_0 \in [a, b]$ such that, h is increasing in $[a, x_0]$ and decreasing in $[x_0, b]$.

single peak property of $x(1 - F(x))$.

2. **Case 2:** $p \leq p' < p''$: In this case, the seller with opportunity cost $c < p'$ joins the D-blocking mechanism conditional on a buyer being present. A buyer with type $v > p''$ joins the D-blocking mechanism if

$$(v - p)Pr(c \leq p|c \leq p') \leq v - p''.$$

As in Case 1 this does not happen if:

$$\forall \, p' > p \; p(1 - G(p)) > (1 - p')G(p'). \tag{8}$$

This is clearly true given the choice of p and the single peak property of $(1 - x)G(x)$.

3. **Case 3:** $p' < p < p''$: In this case no agent will join the D-blocking mechanism.

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Theorem 1 suggests a range of posted prices would be immune to D-blocking. We illustrate with an example. Define the cumulative distribution functions as follows:

$$F(x) = \begin{cases} 1 - \frac{2(x+1)^{\frac{1}{2}} - 2}{x} & x \leq \frac{2}{3} \\ 1 - \frac{3(2(\frac{5}{3})^{\frac{1}{2}} - 2)(1-x)}{x} & x > \frac{2}{3} \end{cases} \qquad G(x) = 1 - F(1 - x)$$

We have $\arg\max_{x \in [0,1]} (1 - x)G(x) = \frac{1}{3}$ and $\arg\max_{x \in [0,1]} x(1 - F(x)) = \frac{2}{3}$. Therefore, any posted price mechanism in the interval $[\frac{1}{3}, \frac{2}{3}]$ is immune to D-blocking. These endpoints of the interval are far apart, one favoring the buyer the other the seller. Therefore, 'equitably' distributing the gains from trade is not essential for stability.

Theorem 1 implies that if the operator of the incumbent mechanism knew the priors over buyer and seller types, they would be unable to propose a stable mechanism unless the priors had the right curvatures. This is confirmed in the next theorem. It can be interpreted as

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saying that immunity to D-blocking obtains when the expected gains from trade for any pair of types is sufficiently large. Therefore, immunity to blocking depends on a joint condition on primitives and the mechanism.

Let $\mathcal{M}$ be any ex-post budget balanced, ex-post individually rational and DSIC mechanism for bilateral trade. Denote by $u_b(v, c)$ and $u_s(v, c)$ the payoff at profile (v, c) of buyer and seller, respectively, under $\mathcal{M}$.

Theorem 2 $\mathcal{M}$ *is immune to D-blocking by a positive spread posted price mechanism if and only if for all $0 \leq y < x \leq 1$ the following holds:*

$$E[u_b(x, c)|c \leq y] + E[u_s(v, y)|v \geq x] \geq x - y. \tag{9}$$

Moreover, when the inequality is violated, at least one of the agents strictly prefers to join the blocking mechanism for all types in the critical set.

Proof.

If for some $0 \leq y < x \leq 1$ inequality (9) is violated, we construct a posted price blocking mechanism. Let $V_b = [x, 1]$ and $V_s = [0, y]$ be the critical set of types for the buyer and the seller, respectively. As inequality (9) is violated, there exists $0 \leq p_1 < p_2 \leq 1$ such that the following holds:

$$E[u_b(x, c)|c \leq y] = x - p_2, \tag{10}$$

$$E[u_s(v, y)|v \geq x] = p_1 - y. \tag{11}$$

For a candidate D-blocking mechanism we choose the positive spread posted price mechanism with prices (p_1, p_2). This mechanism is clearly dominant strategy incentive compatible and budget balanced. We now verify that all types in the critical set weakly prefer the D-blocking mechanism to the mechanism $\mathcal{M}$.

Let $a(v, c)$ be the probability of trade in $\mathcal{M}$ when the the profile of types is (v, c).

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Recall from Myerson and Satterthwaite (1983) that $u_b(\alpha, \beta) = \int_0^\alpha a(t, \beta)dt$ and $u_s(\alpha, \beta) = \int_\beta^1 a(\alpha, t)dt$. Therefore, for all $1 \geq v' \geq x$ and $y \geq c' \geq 0$ the following holds:

$$E[u_b(v', c)|c \leq y] = E[u_b(x, c)|c \leq y] + \int_x^{v'} E[a(v, c)|c \leq y]dv$$

$$\leq E[u_b(x, c)|c \leq y] + (v' - x) = v' - p_2, \tag{12}$$

$$E[u_s(v, c')|v \geq x] = E[u_s(v, y)|v \geq x] + \int_{c'}^y E[a(v, c)|v \geq x]dc$$

$$\leq E[u_s(v, y)|v \geq x] + (y - c') = p_1 - c'. \tag{13}$$

The right-hand side of each inequality in (12) and (13) follows from (10) and (11). Inequalities (12) and (13) ensure that all types in the critical set weakly prefer the D-blocking mechanism to $\mathcal{M}$. It is straightforward to check that when an agent's type is outside the critical set, this agent does not prefer the blocking mechanism to $\mathcal{M}$. If inequality (12) holds with equality for some $v' > x$, then it must be that $E[a(v, c)|c \leq y] = 1$ for all $v' > v > x$. Since $a(v, c)$ is increasing in v, it must be that $a(v, c) = 1$ for all $v > x$ and $c < y$. The same argument applies to the seller as well. If no agent strictly prefers to join the blocking mechanism, it must be that the buyer and the seller trade with positive probability when their types are in their respective critical sets. Incentive compatibility implies that all these types trade at the same price. Hence, $E[u_b(x, c)|c \leq y] = u_b(x, y)$ and $E[u_s(v, y)|v \geq x] = u_s(x, y)$. Budget balancedness implies that $u_b(x, y) + u_s(x, y) \leq x - y$, therefore (9) holds.

To prove the converse we show that if there is a positive spread posted price D-blocking mechanism, inequality (9) is violated for some $0 \leq y < x \leq 1$. Let $0 \leq p_1 < p_2 \leq 1$ be the prices in the D-blocking mechanism and V_b and V_s be the associated critical set of types. As the sets V_b and V_s have positive measure, there exists $x \geq p_2$ and $y \leq p_1$ such that $x \in V_b$ and $y \in V_s$. For all such x the following must hold:

$$E[u_b(x, y)|y \in V_s] \leq E[(x - p_2)I_{\{y \leq p_1\}}|y \in V_s]. \tag{14}$$

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Also, for all $x \notin V_b$

$$E[u_b(x,y)|y \in V_s] \geq E[(x - p_2)I_{\{y \leq p_1\}}|y \in V_s].$$ (15)

The left-hand side of (14) is the expected payoff to the buyer when she participates in $\mathcal{M}$ knowing that the seller's type is in their critical set V_s. The right-hand side is the expected payoff to the buyer when she chooses to participate in the D-blocking mechanism conditional on the seller's type being in the critical set and the seller participating in the D-blocking mechanism. A similar observation yields:

$$E[u_s(v,y)|v \in V_b] \leq E[(p_1 - y)I_{\{v \geq p_2\}}|v \in V_b] \ \forall y \in V_s,$$ (16)

$$E[u_s(v,y)|v \in V_b] \geq E[(p_1 - y)I_{\{v \geq p_2\}}|v \in V_b] \ \forall y \notin V_s.$$ (17)

Expanding both sides of inequality (14) yields:

$$\frac{\int_{c \in V_s} u_b(x,c)g(c)dc}{Pr(c \in V_s)} \leq \frac{(x - p_2)Pr(V_s \cap [0,p_1])}{Pr(c \in V_s)}$$
$$\iff \frac{\int_{c \in V_s} u_b(x,c)g(c)dc}{Pr(V_s \cap [0,p_1])} \leq x - p_2 \iff \frac{\int_{c \in V_s \cap [0,p_1]} u_b(x,c)g(c)dc}{Pr(V_s \cap [0,p_1])} \leq x - p_2$$
$$\iff E[u_b(x,c)|c \in V_s \cap [0,p_1]] \leq x - p_2 \ \forall x \in V_b$$ (18)

Expanding both sides of inequality (15) yields:

$$E[u_b(x,c)|c \in V_s \cap [0,p_1]] \geq x - p_2 \ \forall x \notin V_b$$

Similarly, expanding (16) and (17) yield the following inequalities:

$$E[u_s(v,y)|v \in V_b \cap [p_2,1]] \leq p_1 - y, \ \forall y \in V_s.$$ (19)

$$E[u_s(v,y)|v \in V_b \cap [p_2,1]] \geq p_1 - y, \ \forall y \notin V_s.$$ (20)

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Inequalities (18) and (19) allow us to assume $V_b \subseteq [p_2, 1]$ and $V_s \subseteq [0, p_1]$. Let $x^* = \inf V_b$ and $y^* = \sup V_s$. Since $p_2 > 0$ and $p_1 < 1$, inequalities (18) and (19) imply that $x^* > 0$ and $y^* < 1$. The following equalities hold:

$$E[u_b(x^*, c)|c \in V_s] = x^* - p_2, \tag{21}$$

$$E[u_s(v, y^*)|v \in V_b] = p - y^*. \tag{22}$$

As the distribution of types is atomless, we may assume $x^* \in V_b$ and $y^* \in V_s$.

Note that the payoff to a seller with type $c \in V_s \cap [p_1, 1]$ is zero in the D-blocking mechanism. Therefore, if a seller has type $c \in V_s \cap [p_1, 1]$, it must receive a payoff of zero in $\mathcal{M}$, i.e., almost surely $\forall v \in V_b \ u_s(v, c) = 0$. This is similar to a buyer whose type is in $V_b \cap [p_2, 1]$. Consider the case that $a(x^*, y)$ is constant for all $y \leq y^*$. Ex-post incentive compatibility of the mechanism implies that $u_b(x^*, c) = u_b(x^*, c')$ for any two $c, c' \leq y^*$. In particular, for all $c, c' \in V_s$, $u_b(x^*, c) = u_b(x^*, c')$. It follows from (21) that $u_b(x^*, c) = x^* - p_2$ for all $c \in V_s$. Therefore, for all $c \leq y^*$, $u_b(x^*, c) = x^* - p_2$. Hence,

$$E[u_b(x^*, c)|c \leq y^*] = x^* - p_2. \tag{23}$$

Similarly, if $a(x, y^*)$ is constant for all $x \geq x^*$ we deduce that

$$E[u_s(v, y^*)|v \geq x^*] = p_1 - y^*. \tag{24}$$

Thus, if $a(x, y)$ is constant in the relevant ranges, the proof is complete.

For all $x > x^*$ the following holds:

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$$E[u_b(x, c)|c \in V_c] = E[u_b(x^*, c)|c \in V_c] + \int_{x^*}^{x} E[a(s, c)|c \in V_c]ds$$
$$\leq (x^* - p_2) + (x - x^*) = x - p_2 \tag{25}$$

If inequality (25) holds with equality for any $\bar{x} > x^*$, it must be the case that for all $x > x^*$ and almost all $c \in V_c$, $a(x, c) = 1$. To see why, note that equality for $x = \bar{x}$ implies that $a(x, c) = 1$ for all $x^* < x \leq \bar{x}$. However, $a(\cdot, c)$ is monotone in its first component by dominant strategy incentive compatibility. Therefore, $a(x, c) = 1$ for all $x > x^*$ and $c \in V_c$. Since $a(x, y)$ is non-increasing in y, it must be that $a(x, c) = 1$ for all $x > x^*$ and $y < y^*$. This means that $a(x, c)$ is constant and (23) applies.

Suppose then that inequality (25) is strict for all $x > x^*$. Therefore, $E[u_b(x, c)|c \in V_c] < x - p_2$ for all $x > x^*$. Hence, $x \in V_b$ for all $x > x^*$. A similar argument shows that $y \in V_s$ for all $y < y^*$. This proves the lemma. ∎

3.2   Multilateral Trade

We now allow for more than one buyer and seller.

Theorem 3 *Let $\mathcal{M}$ be an ex-post individually rational, ex-post budget balanced and DSIC mechanism that does not depend on the distribution of types. There is an atomless distribution over types under which $\mathcal{M}$ can be D-blocked by a group of agents. Moreover, at least one of the agents has a strict incentive to join the blocking mechanism.*

Proof. Suppose $\mathcal{M}$ cannot be D-blocked under any atom-less distribution over types. We show that $\mathcal{M}$ must be ex-post efficient. The theorem follows from the fact that such a mechanism does not exist (see Myerson and Satterthwaite (1983)). Consider a profile of valuations $x = (x^b, x^s) \in \prod_{i \in N_b} V_i \times \prod_{j \in N_s} C_j$. Let $I \subseteq N_b$ and $J \subseteq N_s$ be the subset of the sellers and buyer that trade in an efficient allocation. Note that $|I| = |J|$. For each buyer i

let W_i^b be the event that seller types in I are below x_s and the type of buyers in $J \setminus i$ are above $x_{J \setminus i}^b$. Define W_j^s similarly for seller j. Formally,

$$W_i^b = \{(v_{-i}, c) \in \prod_{k \in N_b \setminus \{i\}} V_k \times \prod_{k \in N_s} C_k | \forall k \in N_b \setminus \{i\} \; v_k \leq x_k^b \text{ and } \forall k \in J \; c_k \geq x_k^s \} \text{ and}$$

$$W_j^s = \{(v, c_{-j}) \in \prod_{k \in N_b} V_k \times \prod_{k \in N_s \setminus \{j\}} C_k | \forall k \in N_b \; v_k \leq x_k^b \text{ and } \forall k \in J \setminus \{j\} \; c_k \geq x_k^s \}.$$

If the following inequality is violated one can construct a D-blocking mechanism as in the proof of theorem (2).

$$\sum_{i \in I} E[u_i^b((x_i^b, v_{-i}, c), Q, P) | W_i^b] + \sum_{j \in J} E[u_j^s((v, x_j^s, c_{-j}), Q, P) | W_j^s] \geq \sum_{k \in I} x_k^b - \sum_{k \in J} x_k^s. \quad (26)$$

Inequality (26) must hold for all possible positive density distributions. Consider a sequence of positive density distributions that converge to the distribution that puts probability one on the event that the type profile is x. Therefore, the following must hold:

$$\sum_{i \in I} u_i^b(x, Q, P) + \sum_{j \in J} u_j^s(x, Q, P) \geq \sum_{k \in I} x_k^b - \sum_{k \in J} x_k^s. \quad (27)$$

Inequality (27) implies that the mechanism must be efficient. ∎

Define the distance between two distributions H and H' over types as the integral of $|H - H'|$ over $\prod_{i \in N_b} V_i \times \prod_{j \in N_s} C_j$. Denote this distance by $d(H, H')$. If inequality (26) is violated for some distribution of types, it is also violated by every distribution in a ball around it. Therefore, the set of distributions that satisfy theorem 3 is not 'knife-edge'.

We impose two restrictions on the incumbent mechanism that make D-blocking impossible for all positive density distributions. These restrictions are satisfied by many natural mechanisms, like posted price and the double auction. The first guarantees positive surplus to the highest type buyers and lowest type sellers. Let $n = |N_b|$ and $m = N_s$. Let $((1)_n, (0)_m)$ denote the profile where all buyers have type 1 and all sellers have type 0. For

all, $(v, c) \in \prod_{i \in N_b} V_i \times \prod_{j \in N_s} C_j$ such that $(v, c) \neq ((1)_n, (0)_m)$:

$$0 < p_i^b(v, c) < 1 \text{ for all buyers } i \in N_b \text{ and for all sellers } j \in N_s, \ 0 < p_j^s(v, c) < 1. \quad (28)$$

The second is motivated by a concern for simplicity. Call a mechanism continuous if its ex-post payoffs are continuous in type.

Theorem 4 *Suppose there are at least three agents and assume* $|N_b| \neq |N_s|$. *Let* $\mathcal{M}$ *be a continuous, ex-post individually rational, ex-post budget balanced and DSIC mechanism satisfying (28). For each positive density distribution over types,* $\mathcal{M}$ *can be D-blocked*

Proof.

Suppose, for a contradiction, that no D-block is possible. Without loss of generality, suppose more buyers than sellers. Let v_{-M} be a generic profile of valuations for buyers $m + 1$, $m + 2, ...,$ n. Let $((1)_m, v_{-M}, (0)_m)$ be the profile of types where buyers $1, 2, ..., m$ have value 1 and sellers $1, 2, 3, ..., m$ have opportunity cost 0. If the following inequality holds we identify a successful blocking coalition whose members are buyers $\{1, \ldots, m\}$ and sellers $\{1, \ldots, m\}$:

$$E[\sum_{i=1}^{m} u_i^b((1)_m, v_{-M}, (0)_m), Q, P)] + E[\sum_{j=1}^{m} u_j^s((1)_m, v_{-M}, (0)_m, Q, P)] < m. \quad (29)$$

In inequality (29), the expectation is with respect to v_{-M}. For buyer i let T_i^b be the event that $v_j > x$ for all buyers $j \leq m$ and $j \neq i$, and $c_j < y$ for all sellers j. Define T_j^s similarly. If inequality (29) holds, then continuity of $\mathcal{M}$ implies that we can find numbers $x > y$ with $x - y < 1$ such that:

$$E_{\{v_{\{-i\}}\}}[\sum_{i=1}^{m} u_i^b((x, v_{-i}, c), Q, P)|T_i^b] + E_{\{c_{\{-j\}}\}}[\sum_{j=1}^{m} u_j^s((v, y, c_{-j}), Q, P)|T_j^s] < m(x - y).$$

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Given this, we can construct a D-blocking mechanism as in the proof of theorem 2. Therefore, if a block does not exist, inequality (29) is violated. As there are m buyers, and the maximum total value that can be created in trade is m, it follows that:

$$\text{for almost all } v_{-M}, \ \sum_{i=1}^{m} u_i^b((1)_m, v_{-M}, (0)_m, Q, P) + \sum_{j=1}^{m} u_j^s((1)_m, v_{-M}, (0)_m, Q, P) = m.$$
(30)

Continuity of the payoff functions imply that equality (30) must hold for all v_{-M}. Condition (28) implies that this can only happen if for all $v_{-M} \neq (1)_{n-m}$,

$$\forall 1 \leq i \leq m \ q_i^b((1)_m, v_{-M}, (0)_m) = q_i^s((1)_m, v_{-M}, (0)_m)) = 1$$

$$\forall i > m, \ q_i^b((1)_m, v_{-M}, (0)_m)) = 0$$
(31)

Notice, equation (31) is meaningless unless $m > 1$. Let $\tilde{v} \neq (1)_{n-m-1}$ be a profile of valuations for buyers $m + 2, ..., n$. Condition (31) implies that $q_{m+1}^b((1)_{m+1}, \tilde{v}, (0)_m)) = 0$. Replace the label of buyer $m + 1$ with buyer m; condition (31) implies that $q_{m+1}^b((1)_{m+1}, \tilde{v}, (0)_m)) = 1$ which is a contradiction that implies that a block must exist. ∎

The next result shows that the possibility of a posted price mechanism being immune to D-blocking evaporates once we have multiple buyers and sellers. Call a mechanism that sets the same per-unit price for all agents, i.e., $p_i^b(v, c) = p_j^s(v, c)$ for all buyers $i \in N_b$ and sellers $j \in N_s$ and all $(v, c) \in \prod_{i \in N_b} V_i \times \prod_{j \in N_s} C_j$ a uniform price mechanism. A posted price is an example, but unlike a posted price, a uniform price mechanism can condition its price on the reports of agents.

Theorem 5 *Assume there are at least two buyers and two sellers. Let $\mathcal{M}$ be a continuous uniform price, ex-post individually rational, ex-post budget balanced and DSIC mechanism satisfying (28). For every positive density distribution, there exists a buyer-seller pair who can D-block the incumbent mechanism.*

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Proof.

For a contradiction suppose no block by a pair of agents exists. Consider a buyer i and a seller j. Since the mechanism is uniform price, the maximum total payoff of buyer i and seller j is 1. If a block does not exist, an argument similar to the proof of theorem 4 shows that $q_i^b(1, v_{-i}, 0, c_{-j}) = q_j^s(1, v_{-i}, 0, c_{-j}) = 1$ for all $i \in N_b$ and $j \in N_s$ and all $v_{-i} \neq (1)_{n-1}$ and $c_{-j} \neq (0)_{m-1}$. Let (v, c) be a profile of valuations and opportunity costs such that $v_1 = 1$, $v_i = 0$ for $i > 1$ and $c_1 = c_2 = 0$ and $c_i = 1$ for $i > 2$. We have $q_1^b(v, c) = q_1^s(v, c) = q_2^s(v, c) = 1$. Condition (31) implies that $0 < p(x) < 1$. Ex-post individual rationality implies that $q_i^b(v, c) = 0$ for all $i \geq 2$; and the market does not clear. The contradiction shows that a block by a pair of agents must exist. ∎

A natural mechanism to consider in this context is the Walrasian one. Suppose n buyers and sellers. Let $F_n(p)$ be the empirical distribution of reported buyer types. Similarly, let $G_n(p)$ be the empirical distribution of seller types. Let p^n be the price where $1 - F_n(p) = G_n(p)$. This mechanism is not DSIC. However, as the number of agents increases, it becomes approximately DSIC and we converge to the Walrasian outcome. As the Walrasian outcome is in the core for all type distributions, this appears to contradict the theorems above. It does not. As the number of agents increases, the expected profit of the blocking mechanism will decrease but still be positive. It is only in the continuum limit that the expected profit of a blocking mechanism becomes zero. We interpret this to mean that D-blocking can only take place if the price-taking assumption is violated.

4   Prior Notions of Blocking

Immunity to D-blocking can be interpreted as a notion of the core of a cooperative game of incomplete information. Forges, Minelli, and Vohra (2002) survey various notions of the core for cooperative games of incomplete information.[8] The one closest to D-blocking is the

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[8]The notion of durable decision rules due to Holmstrom and Myerson (1983) is concerned with blocking by the grand coalition only. The same is true of ratifiability due to Cramton and Palfrey (1995).

credible core of Dutta and Vohra (2005).

In the credible core, the incumbent mechanism is any Bayesian incentive compatible mechanism, not just DSIC. Our restriction to DSIC mechanisms is not a bug but a feature. Specifically, Bayesian incentive compatibility is a function of the mechanism and the beliefs of the agents. When a block fails and this event becomes known to the agents in the putative blocking coalition A, it changes their beliefs about the types. Truthful reporting in the incumbent mechanism need not be an equilibrium anymore. Nevertheless, Dutta and Vohra (2005) assume that truth telling continues to be an equilibrium in the incumbent mechanism.[9] This issue does not arise when the incumbent mechanism is DSIC. A second difference is that in Dutta and Vohra (2005) the alternative mechanism is only Bayesian incentive compatible assuming types lie in their respective critical set. It is enforced by barring the participation of types not in the critical set. This is accomplished by choosing 'no-trade' in the event than an agent in A reports a type outside their critical set to the alternative. In our case the alternative mechanism does not rely on such restrictions because it is DSIC.

Dutta and Vohra (2005) is not the last word. We briefly summarize subsequent contributions highlighting differences. Myerson (2007), using the virtual utility construct, proposes a blocking notion that, in addition to the credibility requirements, considers random coalition formation and random allocations for each coalition. Serrano and Vohra (2007) use coalitional voting in an incomplete information environment to incorporate endogenous information transmission among members of a coalition. Finally, Liu et al. (2014) study the implications of common knowledge of stability of a two-sided match when one side of the market has incomplete information about the other side. Unlike them, we define immunity to blocking as a property of an equilibrium of a mechanism, rather than an outcome.

Two other streams of literature also focus on how agents choose between alternative mechanisms. The first is collusion in auctions. Collusion is modeled via an uninformed and

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[9]This is called 'passive beliefs' in Rubinstein (1985); a chosen equilibrium is sustained with prior beliefs off the equilibrium path.

disinterested third party who implements a collusive mechanism. The agents considering collusion are usually set exogenously. Colluding agents, however, are not a blocking coalition in that they continue to participate in the 'incumbent' mechanism. As in our setting the decision to participate or not in a bidding ring reveals private information and different papers have adopted different theories of how to incorporate this. Laffont and Martimort (2000), for example use passive beliefs. Eso and Schummer (2004) allow an agent to update their beliefs given an invitation to collude. Their analysis is limited to the case of just two agents and a fixed auction form. Che and Kim (2009) focuses on designing an auction that is collusion resistant under all possible out of equilibrium beliefs held by bidders. The second stream is 'competing' mechanisms (common agency). The setting is one-sided in that agents are all buyers (or all workers), and they are choosing between alternative mechanisms in which to purchase something. The need for coordination between two sides or budget balance as in our case is absent. See Peters (2014) for a survey.

5 Conclusion

In this paper we focused on incumbent mechanisms that were DSIC. The working paper version of this paper extends these results to the double auction which is one instance of a non-DSIC mechanism. Here we content ourselves with summarizing the difficulties associated with extending the notion of blocking to non-DSIC mechanisms. One could adopt the blocking notion of Dutta and Vohra (2005) if one is prepared to accept the passive beliefs assumptions. However, if one wants the equilibrium of the incumbent mechanism to change so as to be consistent with the knowledge that the block has failed, the precise equilibrium played in the incumbent mechanism will depend upon what information is revealed to each agent. Understanding how different information revelation policies affect blocking is, we think, an interesting question. Among other things, it will help us understand the instability of *any* market including a fragmented one. To see why, suppose an incumbent DSIC mecha-

nism is blocked by an alternative mechanism. By the revelation principle we can mimic this outcome using a composition of the two mechanisms. An agent reports their type and as a function of their report they are assigned to one or the other mechanism. This composed mechanism is ex-post individually rational and ex-post (weakly) budget balanced. However, it need not be DSIC. Whether the composed mechanism can be blocked or not will depend on how ones defines blocking and what information is made available to the agents. Were it DSIC, by our arguments this composite mechanism could be blocked. Were this logic to reproduce itself, it suggests that the only 'stable' outcomes should be a collection of bilateral trades.

These speculations ignore forces like search and transaction costs that would tend towards agglomeration. One of the forces towards agglomeration is arbitrage. One might expect the law of one price would make an ostensibly fragmented market behave like a centralized one. Our model restricts traders to participate in one trade. Traders in the blocking set are not allowed to arbitrage the environment (say, purchase the good in the incumbent mechanism and then sell it in the alternative one, or vice versa). However, one can think of an arbitrageur in our setting as being a blocking mechanism. Suppose an incumbent mechanism, M and an alternative mechanism M'. An arbitrageur who is a buyer in M and a seller in M', is effectively matching a seller in M with a buyer in M'. Now, if some agents had a choice of whether to participate in M or M', we can think of that equilibrium decision as the outcome of a direct revelation mechanism in which agents first report their type and are assigned to M or M'. Hence, as before, we can think of M and M' as being a single mechanism. The arbitrageur, then, is a mechanism that blocks this composite mechanism. Given our discussion of non-dominant strategy incentive compatible mechanisms, whether the arbitrageur can succeed or not depends on which equilibrium agents select in the composite mechanism.

Our model assumed independent private information. Even if one were to allow for common values, based on Peivandi (2013) we expect similar results. That paper shows that

(In the setting it considers) immunity to blocking severely restricts the class of possible mechanisms. Finally, our model has focused solely on instability that arises from the tension between incentive compatibility and budget balance. Search costs and execution risk would be forces that work in the opposite direction..

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